The Modular Structure of Elementary Particles as Revealed by Their Spin, Charge and Mass

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Abstract

In this work we address how eighteen elementary particles, according to the standard model, can be organized as modular networks by taking into account standardized versions of their numeric properties of spin, charge and mass. The coincidence index, based on multiset representation of the extensively used Jaccard index and closely related to the Kronecker delta function, is adopted for obtaining networks of elementary particles whose connectivity can be controlled by a parameter establishing how much the positive and negative similarities are relatively taken into account. The networks, obtained respectively to all 2-by-2 and 3-by-3 combinations of the three particle characteristics, reveal well-defined modules directly related to the main classes and properties of the considered elementary particles. Several interesting results are reported, including the confirmation of the ability of the adopted methodology for obtained network representations of the relationships between the considered elementary particles, as well as several interesting interconnectivity patterns, symmetries, and modular structures observed for the obtained networks. In particular, the network obtained considering all the three particle properties – namely spin, charge, and mass – presented intense symmetry as well as intercalating layers of quarks, leptons and bosons, with the Higgs boson and the graviton resulting in opposite extremities, while the W boson provide a critical connection between two portions of the obtained structure.

> 'Every sentence I utter must be understood not as an affirmation, but as a question.'
>
> *N. Bohr.*

1 Introduction

Humanity's continuing quest to approaching the most fundamental properties of the physical world can be traced back to the classical times and, who knows, even earlier. Nowadays, the large number of theoretical and experimental approaches and results related to elementary particles has been organized in the thus called *standard model* (e.g. [1, 2, 3, 4]) , aimed at understanding and classifying the elementary particles, especially regarding the electromagnetic, weak and strong interactions, typically leaving gravity out.

Though incomplete, the standard model has provided valuable insights about the properties and interrelationships between particles, leading to experimentally verified predictions. Basically, the standard model is a hierarchical construct in which the particles are first divided into two main groups, namely the *elementary fermions* and *elementary bosons*. The former family is then further subdivided into two other subfamilies: *quarks* and *leptons* (with the respective antiparticles).

The elementary bosons are divided into two subfamilies as well: the gauge bosons and scalar bosons. Each of these particles have its specific spin, charge and mass. The elementary fermions have spin 1/2, and the elementary bosons spin 1 (gauge bosons) and 0 (scalar bosons). The quarks are related to strong interactions, the leptons to electroweak interactions, while the gauge bosons are related to all the three forces. From a more theoretical perspective, the standard model can be understood to be underlain mainly by the *quantum field theory*, involving several *symmetry*-related concepts and formulations.

The present work aims at studying the eighteen elementary particles (including the non-observed graviton) in terms of the *similarity* between their respective spin, charge and mass properties, or features. In order to do so, it is necessary to apply some methodology capable of mapping from the table of properties of the elementary particles into respective networks (e.g. [5, 6, 7, 8]).

In the present work, we resource to a recently introduced *coincidence approach* [9] in which multiset theory (e.g. [10, 11, 12, 13, 14, 15]) is used to express a real-valued version of the extensively used Jaccard similarity index (e.g. [16, 17]), which has been shown [18] to be directly related to the Kronecker delta function. The coincidence similarity index corresponds to the produce of the real-valued Jaccard and overlap (e.g. [19]) indices, so as to allow the relative *interiority* between the two compared sets of real-valued measurements to be taken into account. In addition, a parameter α provides an effective means for controlling the obtained interconnections.

Basically, the three properties of the elementary particles are first standardized into non-dimensional counterparts with null mean and unit standard deviation, the the similarities between the property of each pair of particles is then determined, yielding the respective weight matrix defining the respectively obtained networks, which can then be visualized and analyzes by using respective methodologies (e.g. [7]). In the present work we adopt the Fruchterman-Reingold visualization method [20].

There are at least two main motivations for developments reported in the present work. First, it is interesting to investigate to which point the sole consideration of the similarities between the elementary particles properties can lead to modular structures potentially related to the overall families in the standard model. In particular, a good adherence between the obtained modules and the standard model structure could be understood as corroborating the fact that the only three respective properties are indeed decisive in establishing the main relationships between the considered particles. In addition, by varying the combinations of properties taken into account, which is done in 2-by-2 and 3-by-3 manner, it is also possible to identify the respective effect of each of these combinations regarding relationships with the standard model structure.

The second main motivation of this work is to illustrate the potential of the coincidence methodology regarding a simple, but very important dataset, which involves properties of rather different types and magnitudes. As such, the reported approach and results illustrate a more general procedure that can be applied to other important problems in physics and chemistry, such as studying other systems, such as the periodic table, in terms of the similarity of the involved properties.

We start by presenting the concepts and methodology underlying the coincidence approach to building networks from tables of measurement, and then present the adopted standardization procedure. The several elementary particles obtained for distinct combinations of spin, charge and mass respectively to several interconnectivity levels are then presented and discussed.

2 The Coincidence Approach

Though the Kronecker delta function can be understood as providing the most direct and strict test of similarity between two real values [18], this quantification is just too strict in the sense that any small error in a pair of otherwise identical values will lead to null result.

Several similarity indices, including the extensively used Jaccard similarity (e.g. [16, 17], as well as the cosine (e.g. [10]), Pearson (e.g. [21]) and L1-related similarity indices (e.g. [22, 23, 24]) have been reported in the literature. Interestingly, several of them can be understood [18] as being related to the Kronecker delta function, though providing a more tolerant quantification of pairwise similarity. In addition, several of these indices have been related to multiset theory generalized to take into account real, possibly negative values [9, 18].

The present work focuses on the *coincidence similarity* [9], which is related to both the real-valued Jaccard and overlap indices. More specifically, given that the former of these indices has been shown not to be able to take into account the relative interiority of the two compared sets, multisets, functions or fields, the product of the real-valued Jaccard and interiority index provides an interesting manner to integrate the similarity information provided by these two indices.

In order to provide selection of the overall degree of interconnections in the resulting networks, we resource to a controlling parameter α, with $0 \leq \alpha \leq 1$. The higher the value of this parameter, the most the positive pairwise correlations are taken into account by the similarity quantification.

The adopted coincidence similarity index can be expressed as:

$$\mathcal{C}_R(\vec{x}, \vec{y}, \alpha) = \mathcal{J}_R(\vec{x}, \vec{y}, \alpha) \; \mathcal{I}_R(\vec{x}, \vec{y}) \tag{1}$$

where $\mathcal{I}_R(\vec{x}, \vec{y})$ is the *real-valued interiority index* given as:

$$\mathcal{I}_R(\vec{x}, \vec{y}) = \frac{\sum_{i \in S} \min\{|x_i|, |y_i|\}}{\min\{S_x, S_y\}} \tag{2}$$

where S is the support of the vectors ($i = 1, 2, 3$ in the case of elementary particles), with:

$$S_x = \sum_{i \in S} |x| \; dx; \quad S_y = \sum_{i \in S} |y| \; dy; \tag{3}$$

In addition, $\mathcal{J}_R(\vec{x}, \vec{y})$ is the α-controlled *real-valued Jaccard index* [25], which can be expressed as:

$$\mathcal{J}_R(\vec{x}, \vec{y}, \alpha) = \frac{s_{\pm}(\vec{x}, \vec{y}, \alpha)}{\sum_{i \in S} \max\{|x_i|, |y_i|\}} \tag{4}$$

where:

$$s_{\pm}(\vec{x}, \vec{y}, \alpha) = [\alpha] \, s_{+}(\vec{x}, \vec{y}) - [1 - \alpha] \, s_{-}(\vec{x}, \vec{y}) \tag{5}$$

with:

$$s_+(\vec{x},\vec{y}) = \sum_{i \in S} |sign(x_i) + sign(y_i)| \min \{|x_i|, |y_i|\} \quad (6)$$

$$s_-(\vec{x},\vec{y}) = \sum_{i \in S} |sign(x_i) - sign(y_i)| \min \{|x_i|, |y_i|\} \quad (7)$$

Expression 6 appeared previously [22] in the context of L1 spaces. This expression incorporates the same sign pairwise contributions to the sum, while Expression 7 collects the opposite sign cases. The parameters α therefore controls a linear combination of these two effects. The multiset-based real-valued Jaccard index [9, 18] is obtained when $\alpha = 0.5$ (see also [23] in the context of L1 similarity). However, for $\alpha > 0.5$ the same signal pairwise relationships will predominate and the similarities will become stronger. Consequently, it becomes possible to control the overall level of interconnectivity of the respectively obtained network representations of the original data.

3 Features Standardization

Table 1 presents the eighteen elementary particles studied in this work in terms together with their respective spin, charge and mass (e.g. [26]).

id.	*particle*	*spin*	*charge*	*mass*
1	up quark	1/2	2/3	2.2
2	down quark	1/2	−1/3	4.6
3	charm quark	1/2	2/3	1280
4	strange quark	1/2	−1/3	96
5	top quark	1/2	2/3	173100
6	bottom quark	1/2	−1/3	4180
7	electron	1/2	−1	0.511
8	electron neutrino	1/2	0	0.0000022
9	muon	1/2	−1	105.7
10	muon neutrino	1/2	0	0.17
11	tau	1/2	−1	1776.86
12	tau neutrino	1/2	0	15.5
13	photon	1	0	0
14	W boson	1	−1	80.385
15	Z boson	1	0	91.1875
16	gluon	1	0	0
17	Higgs boson	0	0	125.00
18	graviton	2	0	0

Table 1: The eighteen considered elementary particles and their respective spin, charge and mass values. In this work, quarks will be shown in light blue, leptons in salmon, boson in green, and the graviton in magenta.

In several theoretical and practical situations, we have that the involved measurements or features present rather distinct ranges of magnitudes, an issue that also depend on the adopted units. The case of the considered properties of elementary particles is no exception, especially regarding the mass magnitudes.

Discrepant magnitudes among a set of measurements can be addressed by applying the statistical procedure known as *standardization* (e.g. [21, 27, 28]). which consists in applying the following expression to each of the measurements:

$$\tilde{x_i} = \frac{x_i - \mu_{x_i}}{\sigma_{x_i}} \quad (8)$$

The obtained measurements can be verified to correspond to non-dimensional quantities with null means and unit standard deviation, most of which resulting comprised in the interval $[-2, 2]$.

Figure 1 illustrates the properties of each of the eighteen considered elementary particles after respective standardization. Observe the effectiveness of the standard procedure even respectively to the wide mass variations.

4 Networks of Elementary Particles

The coincidence approach to transform a table of elements into a respective network was applied to the elementary particles in Table 1 while taking distinct combinations of the properties spin, charge and mass. Three interconnections levels were obtained by making $\alpha = 0.4, 0.5, 0.6$, while the overall threshold for taking into account the pairwise similarities was fixed at 0.35.

Figure 2 presents the networks obtained while considering the *spin* and *charge*, respectively to $\alpha = 0.4$ (a), 0.5 (b), and 0.6 (c). For $\alpha = 0.4$, in which case only the strongest similarities are maintained, we can discern 5 modules (or communities), each composed by exactly 3 particles: two of them are quarks, two are leptons, and the remainder one are bosons. The W boson, Higgs boson, and the graviton resulted isolated at this level of interconnectivity. When α was increased to 0.5, the boson triple remained stable, but the other modules merged pairwise in a symmetric manner, while respectively absorbing the W and H bosons. Interestingly, most of the resulting communities (or modules) involve particles from distinct groups. For the most interconnected configuration, implied by $\alpha = 0.6$, the boson triple formed a clique with the graviton, while the other two modules remained stable. The obtained modules therefore reflect the similarities between the considered elementary particles as far as the spin and charge properties are concerned, which makes physical sense.

Figure 3 illustrates the elementary particle networks obtained while considering the *spin* and *mass* respectively

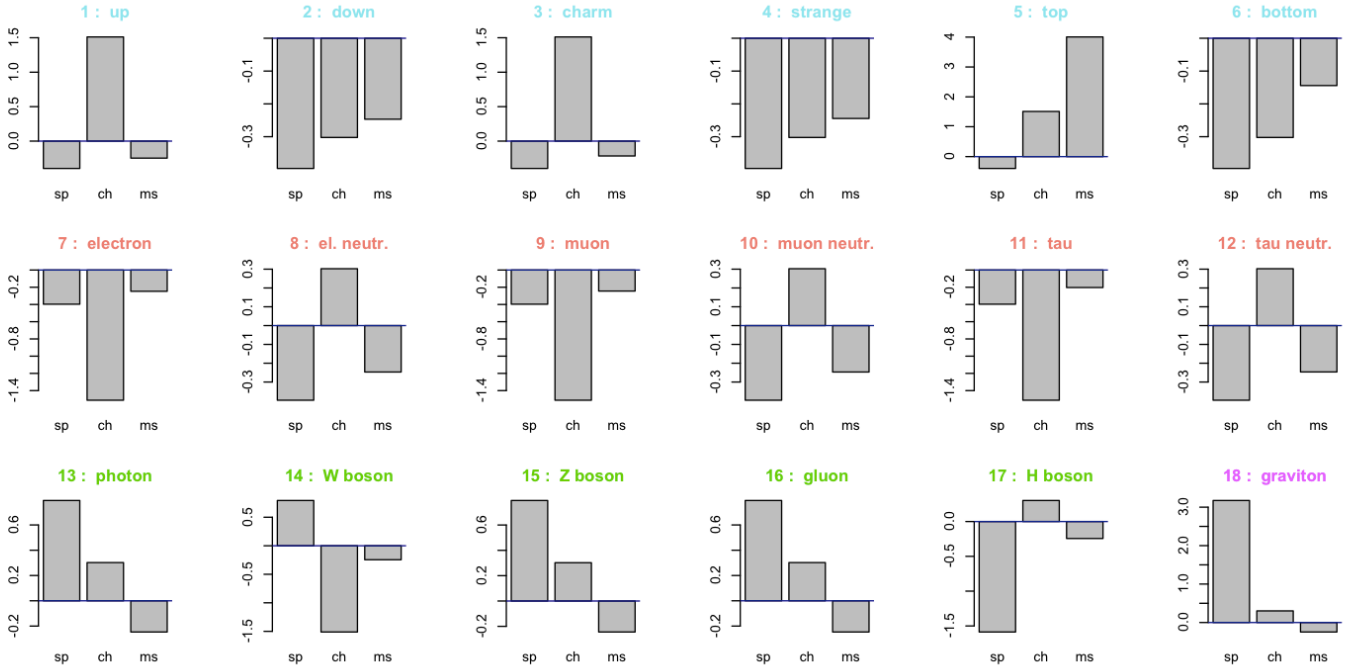
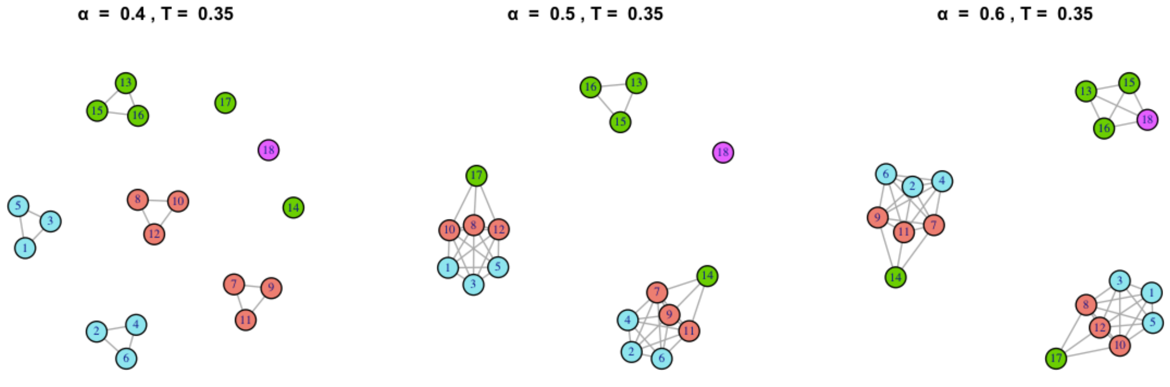


Figure 1: The eighteen elementary particles considered in the present work in terms of their statistically standardized spin, charge and mass. Each of the new measurement is a non-dimensional quantity with null mean and unit standard deviation, most of which comprised in the interval $[-2, 2]$. The standardization is performed considering all particles, but only one property at a time. The obtained standardize features are considered for obtaining the pairwise coincidence similarities adopted in the approach reported in the present work, allowing the respectively obtained networks to be visualized.



(a) (b) (c)

Figure 2: The networks of elementary particles obtained for *spin* and *charge* for interconnectivity levels $\alpha = 0.4, 0.5, 0.6$. It is interesting to consider the obtained interconnections in terms of the similarities between the standardized measurements shown in Fig. 1.

to $\alpha = 0.4$ (a), 0.5 (b), and 0.6 (c). Identical networks were obtained for $\alpha = 0.4$ and 0.5, being characterized by a large module with 11 particles, most of which light and corresponding to either quarks or leptons (therefore with spin 1/2), plus another medium sized community containing four bosons. The top quark, Higgs boson and the graviton resulted in isolated nodes. As α was changed to 0.6, the graviton merged into the bosons module, while the top quark remained isolated. These relationships are indeed reasonable as far as the combination of spin and mass are concerned.

The networks of elementary particles obtained while taking into account the *charge* and *mass* are shown in Figure 4 respectively to $\alpha = 0.4$ (a), 0.5 (b), and 0.6 (c). Now, the combination of the charges and masses in defining the pairwise similarity between the elementary particles resulted in a large community for $\alpha = 0.4$, which contains mostly light particles with null charge. The sec-

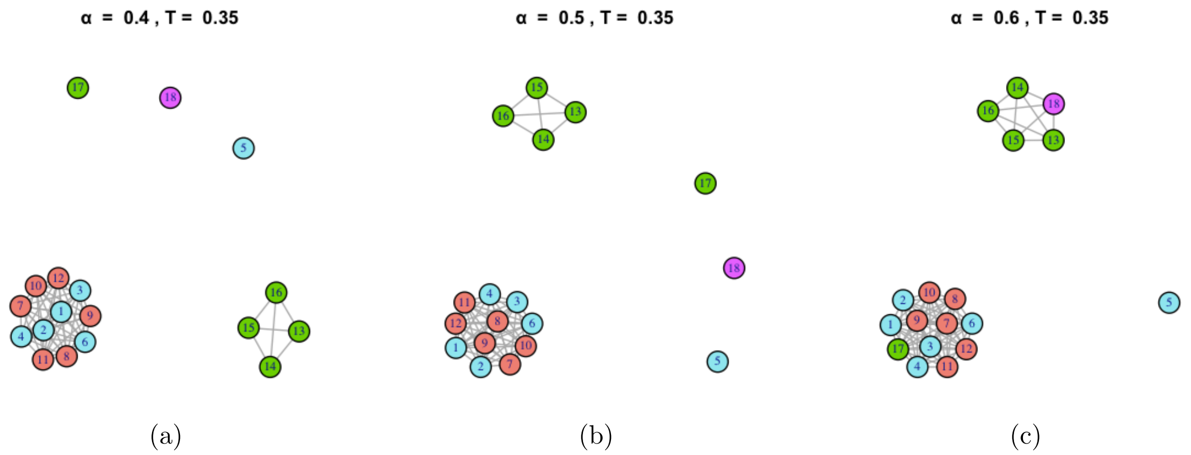


Figure 3: The networks of elementary particles obtained for *spin* and *mass* for interconnectivity levels $\alpha = 0.4, 0.5, 0.6$.

ond largest module incorporates 4 particles, all with negative charge, including the W boson. Two minor modules of quarks with similar charges can also be observed, while the top quark resulted separated as a consequence of its large mass. The same modules can be observed for $\alpha = 0.5$, but for $\alpha = 0.6$ the two smaller quark communities are respectively absorbed by the larger modules, while the top quark remains isolated. These are the modular structures mediated by the charge and mass, which again make sense physically, revealing another perspective from which the particle relationships can be observed.

Figure 5 depicts the elementary particle networks obtained while considering all the available three properties of spin, charge and mass, respectively to $\alpha = 0.4$ (a), 0.5 (b), and 0.6 (c). This result is of particular interest because it takes into account all the three particle properties, therefore providing the most complete description of the modularities mediated by these measurements. Three modules can be clearly discerned for $\alpha = 0.4$. Two of them, with 7 and 6 nodes, are almost perfectly symmetrical combinations of quarks and leptons, plus a respective boson. The quark that is missing from the module with 6 nodes is very probably the top quark, which again resulted isolated. A smaller community with 4 bosons can also be observed, as well as the isolated graviton and Higgs boson.

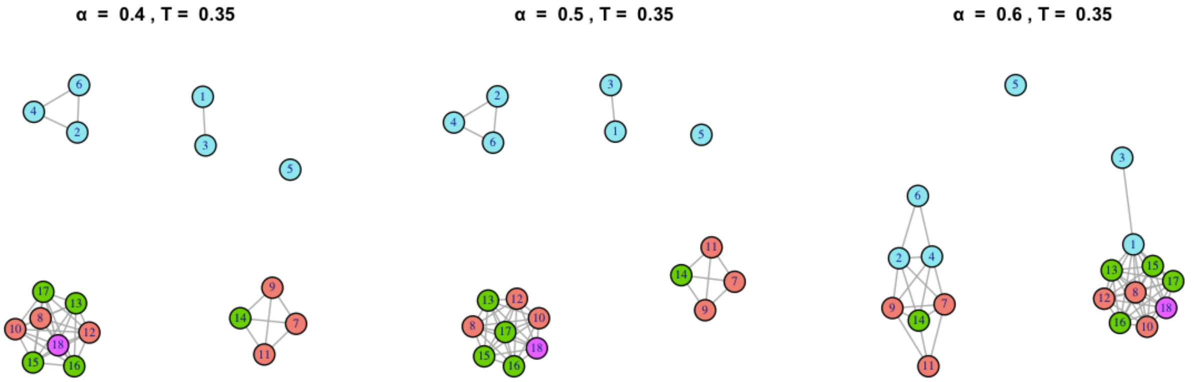
Interestingly, the two largest communities can be observed to merge for $\alpha = 0.5$, while the graviton is absorbed into the boson community and the top quark remain separated. Then, for $\alpha = 0.6$, all the particles coalesce into a single connected component in which layers of quarks, leptons and bosons alternate in a mostly symmetric manner, with the graviton and Higgs boson resulting in opposite extremities. Observe also the top quark finally being connected to the other related quarks in the lower portion of the obtained network. It is also interesting to observe the role of the W boson regarding the interconnection with the upper portion of the obtained network, as well as the quite distinct interconnectivity of the Higgs boson appearing in the opposite portion as of the other bosons. The unique perspective of the graviton is reflected in its extreme position.

5 Concluding Remarks

Science has been built upon an eclectic theoretical and experimental framework in which a diversity of approaches have been systematically employed in order to obtain insights about the structure and dynamics of the physical world. This include the consideration of symmetries, distances, and similarities, which have been applied both as insights motivating experimental approaches as in the interpretation of experimental data.

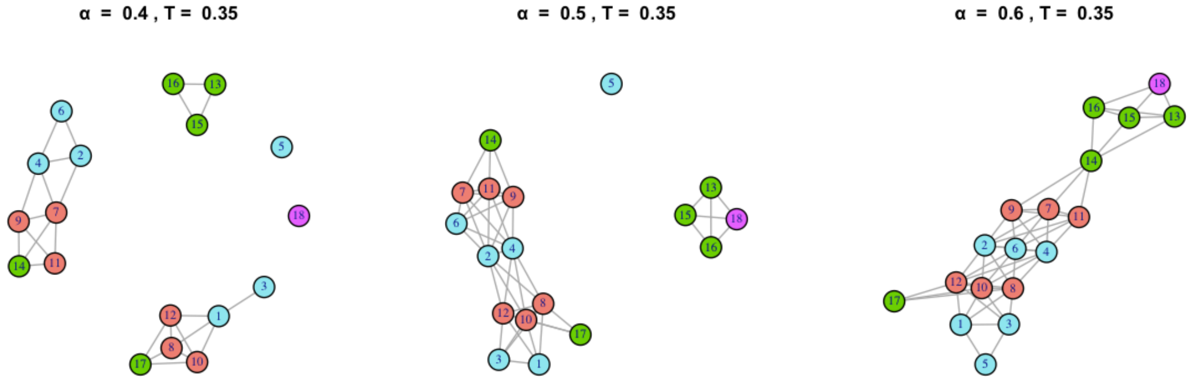
The present work has studied the interrelationship between eighteen elementary particles from the perspective of pairwise similarities between their respective properties of spin, charge and mass. The fact that these properties, especially the mass, have different magnitude was approached by adopting the statistical linear transformation known as standardization. In addition, we applied a recently developed methodology based on the coincidence similarity index that allows networks to be obtained from tables of measurements. The coincidence index is directly related to the Jaccard similarity index, complemented by the incorporation of information about the relative interiority of the two compared sets, as well as to the Kronecker delta function, of which it can be understood as providing a more tolerant quantification of similarity, though not as much as the cosine and the Pearson similarities that have been extensively adopted.

Networks with detailed interconnectivity and well-



(a) (b) (c)

Figure 4: The networks of elementary particles obtained for *charge* and *mass* for interconnectivity levels $\alpha = 0.4, 0.5, 0.6$.



(a) (b) (c)

Figure 5: The networks of elementary particles obtained while taking into account *spin*, *charge* and *mass* for interconnectivity levels $\alpha = 0.4, 0.5, 0.6$.

defined modular structures have been obtained from datasets by using the coincidence approach [25]. In addition, the provision of a parameter α controlling the overall level of interconnectivity provides a particularly useful dissecting capability that can be employed to verify how the identified structures and modules change regarding more or less intense interconnections. More specifically, as the values of α are progressively reduced, only the stronger links remain that are capable of counteracting the opposite sign interactions. Thus, more intensely interconnected networks are obtained for larger values of α. These features contributed substantially for the analysis of the elementary particle networks obtained and discussed in the current work.

We were therefore able to obtain several networks representing the similarity between the considered eighteen elementary particles, while considering all the 2-by-2 and 3-by-3 combinations of the properties spin, charge and mass. In each case, networks were obtained that reflected the respectively adopted measurements, often resulting in symmetric interconnectivity patterns and modules, such as the networks observed when taking into account the spin and charge. The networks obtained while considering the weight resulted in a particularly diverse sizes of modules. When all the three properties were considered, three main communities could be identified corresponding to two mostly symmetric modules involving almost the same number of quarks and leptons, but just one boson, plus a third community containing only three bosons. As the nodes were allowed to develop more intense interconnectivity, which was obtained by considering larger values of α, the two mixed communities merged in a mostly symmetric, laminated module with alternating layers of quarks and leptons, with a boson at each of its extremities. As α was further increased, a single network was obtained incorporating all considered particles, being organized by alternating layers of quarks, leptons and bosons, while the Higgs boson and the graviton resulted in opposite extremities. Among the reported results, this latter network represents the most complete representation of the interactions between the several considered elementary particles as far as their respective pairwise similarity of properties are concerned.

The obtained results should be understood as being basically aimed at a systematic approach to verifying the possible implications of the pairwise similarity between the three properties of the considered particles in shaping their respective relationships. The basic question, therefore concerns how the elementary particles group themselves into specific interconnectivity patterns and modularity as a consequence of similarity relationships between their respective spin, charge and mass, and how the obtained community structures can be related to the structure of the standard model. After all, it is these three properties that characterized the behavior of the elementary particles. However, it is by no means meant that the obtained modularity should be closely mirrored in the standard model, as other aspects, such as the types of mediated forces, have particular importance in the latter.

All in all, the proposed concepts and methods could provide flexible subsidies for exploring several natural systems in terms of similarity relationships established as a consequence of distinct choices of the features adopted for respective characterization, providing insights that can then be further investigated through theoretical or experimental means.

Several perspectives for further research are motivated by the reported concepts, methods and results, including the extension to other systematic models such as the periodic table and other types of composite particles and physical interactions.

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References

[1] R. Oerter. *The Theory of Almost Everything: The Standard Model, the Unsung Triumph of Modern Physics*. Plume, 2006.

[2] B. A. Schumm. *Deep Down Things: The Breathtaking Beauty of Particle Physics*. Johns Hopkins University Press, 2004.

[3] G. D. Coughlan, J. E. Dodd, and B. M. Gripaios. *The Ideas of Particle Physics: An Introduction for Scientists*. Cambridge University Press, 2006.

[4] D. J. Griffiths. *Introduction to Elementary Particles*. John Wiley and Sons, 2006.

[5] A.L. Barabási and Pósfai M. *Network Science*. Cambridge University Press, 2016.

[6] M. Newman. *Networks: An Introduction*. Oxford University Press, 2010.

[7] L. da F. Costa, F. A. Rodrigues, G. Travieso, and P. R. Villas Boas. Characterization of complex networks: A survey of measurements. *Advances in Physics*, 56(1):167–242, 2007.

[8] L. da F. Costa, O.N. Oliveira Jr., G. Travieso, F.A. Rodrigues, P.R. Villas Boas, L. Antiqueira, M.P. Viana, and L.E.C. Rocha. Analyzing and modeling real-world phenomena with complex networks: a survey of applications. *Advances in Physics*, 60(3):329–412, 2011.

[9] L. da F. Costa. Further generalizations of the Jaccard index. https://www.researchgate.net/publication/355381945_Further_Generalizations_of_the_Jaccard_Index, 2021.

[10] J. Hein. *Discrete Mathematics*. Jones & Bartlett Pub., 2003.

[11] D. E. Knuth. *The Art of Computing*. Addison Wesley, 1998.

[12] W. D. Blizard. Multiset theory. *Notre Dame Journal of Formal Logic*, 30:36—66, 1989.

[13] W. D. Blizard. The development of multiset theory. *Modern Logic*, 4:319–352, 1991.

[14] P. M. Mahalakshmi and P. Thangavelu. Properties of multisets. *International Journal of Innovative Technology and Exploring Engineering*, 8:1–4, 2019.

[15] D. Singh, M. Ibrahim, T. Yohana, and J. N. Singh. Complementation in multiset theory. *International Mathematical Forum*, 38:1877–1884, 2011.

[16] P. Jaccard. Distribution de la flore alpine dans le bassin des dranses et dans quelques régions voisines. *Bulletin de la Société vaudoise des sciences naturelles*, 37:241–272, 1901.

[17] Wikipedia. Jaccard index. https://en.wikipedia.org/wiki/Jaccard_index. [Online; accessed 10-Oct-2021].

[18] L. da F. Costa. On similarity. https://www.researchgate.net/publication/355792673_On_Similarity, 2021.

[19] M. K. Vijaymeena and K. Kavitha. A survey on similarity measures in text mining. *Machine Learning and Applications*, 3(1):19–28, 2016.

[20] T. M. Fruchterman, M. J. Thomas, and E. M. Reingold. Graph drawing by force-directed placement. *Software Practice and Experience*, 21:1129–1164, 1991.

[21] R. A. Johnson and D.W. Wichern. *Applied multivariate analysis*. Prentice Hall, 2002.

[22] B. Mirkin. *Mathematical Classification and Clustering*. Kluwer Academic Publisher, Dordrecth, 1996.

[23] C. E. Akbas, A. Bozkurt, M. T. Arslan, H. Aslanoglu, and A. E. Cetin. L1 norm based multiplication-free cosine similiarity measures for big data analysis. In *IEEE IWCIM*, France, Nov. 2014.

[24] C. E. Akbas, A. Bozkurt, A. E. Cetin, R. Cetin-Atalay, and A. Uner. Multiplication-free neural networks. In *Signal Processing and Communications Applications Conference (SIU)*, Malatya, Turkey, May. 2015.

[25] L. da F. Costa. Coincidence complex networks. https://www.researchgate.net/publication/355859189_Coincidence_Complex_Networks, 2021.

[26] Wikipedia. Standard model. https://en.wikipedia.org/wiki/Standard_Model, 2021.

[27] R. O. Duda, P. E. Hart, and D. G. Stork. *Pattern Classification*. Wiley Interscience, 2000.

[28] K. Koutrombas and S. Theodoridis. *Pattern Recognition*. Academic Press, 2008.